THE EFFECT OF FREQUENCY UPON THE CORONA

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THE ELECTRIC STRENGTH OF AIR—V
The Influence of Frequency

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ABSTRACT OF PAPER

The paper presents the account of an investigation of the influence of frequency on the corona, between 60 and 3000 cycles per second. The wire and coaxial cylinder method was used.

A simple method for measuring the maximum value of an alternating voltage wave has been developed and used for obtaining this maximum value at the critical corona voltage. The gold-leaf electroscope was used to detect the first appearance of corona.

It has been found that for frequencies above 275 cycles per second the corona voltage (maximum value) is not so uniform and constant as it is at frequencies in the neighborhood of 60 cycles.

The indications of the investigation are that at 2000 cycles the corona voltage is lower than it is at 60 cycles by about 3 or 4 per cent.

The experiments reveal several interesting instances of resonance phenomena in the high-tension transformer circuits.

IN AN earlier paper by one of the authors[1] it was shown that within the range 20 to 90 cycles per second the frequency had a measureable effect on the voltage (maximum of the wave) at which the corona begins. The principle of the method of experiment is simple and will be repeated briefly here. The corona is formed about a wire which is stretched coaxially in a metal cylinder, the ends of the cylinder being closed by insulating bushings. Air may be let into the cylinder by a lateral tube near one end and out by a lateral tube near the other end. Immediately after leaving the cylinder the air passes over an insulated electrode which is connected to a gold leaf electroscope. The walls of the cylinder where the lateral tubes were connected were drilled with numerous small holes. The air could thus pass freely and at the same time the electrode could not be influenced electrostatically by the presence of the highly charged wire at the axis of the cylinder. As the voltage applied to the wire

1. Whitehead, TRANS. A.I.E.E., 1910, Vol. XXIX, Part II, p. 1159; and 1911, Vol. XXX, Part III, p. 1857.

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is gradually raised a value is reached at which the air around the wire is ionized, that is, becomes conducting. The electroscope, as soon as this occurs, begins to lose its charge. The cylinder was always connected to earth.

It was found that the point at which the electroscope began to leak was the same whether the air was moving through the tube or whether the air was at rest, and that initial ionization and the visual corona were always contemporaneous. It was shown too, that for a given size wire and tube under the same conditions of temperature and pressure, the corona always began when the voltage in its cycle reached a definite value and that this value was approximately the same as the maximum value of the voltage at which the electroscope first began to leak. Another result was that the electric field strength at the surface of wire when corona began was independent of the size of the tube.

The value of the voltage delivered by the high-tension side of the transformer was determined from the ratio of transformation and from oscillograms taken of the voltage impressed across the low-tension terminals of the transformer. It was assumed that the wave shape of the high-tension e.m.f. was the same as that of the low-tension e.m.f.

The uniform appearance of the corona at a definite value of voltage regardless of position on the wave would seem to indicate that the corona voltage should be independent of the frequency. This, however, was not borne out by the experiments. The corona voltage was found to decrease with increasing frequency. A curved plotted between frequency and the maximum value of the voltage wave for the critical voltage (*i.e.*, beginning of corona) showed a fairly regular decrease from 20 cycles to 90 cycles. This decrease had the same percentage value for the various sizes of wire tried. The voltage at 90 cycles was about 6 per cent lower than the voltage of 20 cycles.

The influence of frequency has been studied by comparatively few investigators, and far from completely. Peek[2], in the range 40 to 100 cycles per second, was unable to detect any influence of frequency either on the voltage at which the visual corona started or on the voltage at which the loss began. Very recently Alexanderson[3] has found that at 100,000 cycles per second, a 3-in. (76.2-mm.) spark gap between spheres 5 in. (127 mm.) in diameter breaks down at approximately the same voltage (100,000 volts)

2. Peek, TRANS. A. I. E. E., 1912, Vol. XXXI, Part I, p. 1055.
3. Alexanderson, *General Electric Review*, page 427, April 1914.

that it does at commercial frequencies; a 5-in. gap between plane parallel plates, however, was found to break down at 30,000 volts—a much lower value than the breakdown voltage for commercial frequencies. Recent results of Weidig and Jaensch[4] indicate that for increasing values of voltage above the corona-forming voltage, the corona starts at a lower and lower and ends at a higher and higher instantaneous value; this is at variance with the observations reported in an earlier paper of this series and referred to above.

The work of the present paper was undertaken in order to extend the observations of the corona voltage over a wider range of frequency.

Experimental Apparatus

The generator available, although structurally a unit (direct-driven) consisted in fact of two generators; one having 48 poles and the other 240 poles. At the rated full speed, 1500 rev. per min., one generator gave 600 cycles and the other 3000 cycles per second. Each machine was rated at 110 volts, 5 amperes. Two transformers were used, one rated at 500 watts, 100-10,000 volts at 133 cycles per second, and a second, employed in the larger part of the work, rated at 3000 watts at 60 cycles. The low-tension winding of the latter transformer was in two 50-volt sections, the high-tension winding was in four sections, the rated voltage per section being 6250 volts.

The high-frequency generator was driven by a d-c. 240-volt shunt motor. The field of the generator was supplied from a 120-volt storage battery. The large amount of inertia possessed by the revolving field of the generator prevented any sudden changes in speed due to changing voltage of the direct-current supply. The conditions were thus favorable to steadiness in the readings. The speed of the generator was controlled partly by the use of resistance in the motor armature circuit and partly by varying the resistance of the motor field circuit.

Speeds from normal to three-quarters normal were obtained by the use of armature resistance; speeds from three-quarters to half normal were obtained by running the motor armature from the 120-volt mains and by the use of resistance in the field circuit. It was not feasible to run the machine at any speed much below half speed, because of the large field current required by the generator, and because of speed fluctuations. Control of

4. Weidig and Jaensch, *Electrotechnische Zeitschrift*, Heft 23 and 24, 1913.

the transformer voltage was effected by varying the field current of the generator.

The corona apparatus proper consisted, as in the earlier experiments, of a wire stretched coaxially in a metal tube. At a point equidistant from the ends of the tube were drilled numerous small holes (about $\frac{1}{8}$ in. (3.2 mm.) in diameter). The electrode, which consisted of a brass disk having the same curvature as the tube, was placed over these holes and as near the tube as could be conveniently be done without having metallic contact. A grounded metallic shield was placed around the electrode and the connection to the electroscope was made through a hole in the shield, the wire being insulated from the shield by means of a plug of sulfur. The electroscope was a very sensitive one; it gave a workable deflection of the gold-leaf when charged to a potential of 120 volts. In the earlier part of the work the ends of the tube were left open, the wire being stretched between insulating supports situated at some distance from the ends of the tube. Later the ends of the tube were closed by insulating caps, the wire passing through holes in the caps.

The Experiments

Observations were first made on a brass wire 0.089 cm. (0.0350 in.) in diameter in a 4.9-cm. tube. The 500-watt transformer mentioned above was used. The procedure was to raise the primary voltage by increasing the field current of the generator until the electroscope began to discharge. The field current was then decreased until the leak of the electroscope stopped, and then increased again. The voltage at the instant the leak began was read by a hot-wire voltmeter, connected across the low-tension terminals of the transformer. The speed was read by a carefully calibrated tachometer. No attempt was made in these experiments to determine the wave shape, this being a matter of some difficulty at the frequencies in question. In the neighborhood of 600 cycles, the electroscope began to leak at about 80 volts, at 1400 cycles at 73 volts and finally, at 2800 cycles, at 37 volts. The decrease in voltage between 1400 and 2800 cycles was fairly regular.

It seemed highly improbable that the corona voltage actually experienced as great a change as was indicated by the above figures. To test the matter further a high-tension electrostatic voltmeter was connected across the high-tension terminals of the transformer.

It was found that the corona voltage as indicated by the static voltmeter remained constant. The corona tube was then disconnected from the transformer and at various frequencies the voltage was adjusted so that this voltmeter always read 10 kilovolts. At 60 cycles the low-tension voltage was 82.5 volts, and at 2750 cycles, 31 volts. It was thus seen that the ratio of transformation of this transformer was *over two and one-half times* as great at 2750 cycles as it was at 60 cycles. The cause of this great change is evidently the capacity of the high-tension windings. The charging current taken by the high-tension windings, other things being equal, is directly proportional to the frequency. The reactance of these same windings is also directly proportional to the frequency. Consequently one would expect the disturbing effect of the charging current on the ratio of transformation to be, roughly, directly proportional to the square of the frequency.

That the current taken by the transformer was leading was easily shown. An electrodynamometer connected in the low-tension side indicated 5.5 amperes at 2750 cycles—ten times the value of the current taken at 60 cycles for the same high-tension voltage. The large reactance of this instrument (22 ohms at 2750 cycles) was in this instance of great advantage, because by applying the three-voltmeter method to the dynamometer and transformer the relative power factor of the two could be at once determined. The power factor of the dynamometer alone was determined by series connection with incandescent lamps (assumed non-inductive) and applying the same method. It was found that the current taken by the transformer at 2750 cycles was leading the voltage by about 60 degrees.

For further study of this disturbing influence tests were made with the 3000-watt transformer described above. The high-tension voltage was not measured, but the effect of different methods of connection of the high-tension coils was studied. All the experiments were conducted at about 2750 cycles. A few examples of the results may prove of interest. With all the high-tension windings in series, one end of the winding connected to ground and the other end open, the transformer took 3.1 amperes at 3.5 volts, the current leading the voltage by about 20 degrees. With the high-tension windings connected two in series, two in parallel, the same point being earthed as formerly, the transformer took 4.0 amperes at 33.5 volts, the current leading the voltage by about 90 degrees. In both cases the low-

tension windings were in series. These two cases are fair samples of the results found in the course of the experiments, and indicate that the method of connecting the high-tension windings has a pronounced influence on the readings of the instruments in the primary side of the transformer.

There was also evidence that at one frequency within the range studied, the transformer was in resonance. With a certain arrangement of the high-tension windings and running at 2750 cycles the switch of the driving motor was opened. With the resulting decrease of speed and frequency, the voltage applied to the transformer and the current taken by it both began to increase and kept on increasing until it was necessary to open the transformer circuit in order not to injure the instruments. There were also indications that the principal factor in these effects was the capacity between winding and core and not beteen successive portions of the winding. It was found, for example, that the readings on the low-tension side of the transformer depended only upon the maximum voltage (from ground) reached in the high-tension winding. The effect produced by connecting all the coils in series and grounding the middle point of the winding was the same as that produced by connecting the windings two in series and two in parallel and grounding one end.

It appears probable from the results of this paper that the 6 per cent lowering of the corona voltage in going from 20 to 100 cycles reported in the second of this series of papers may be explained as a disturbance due to the capacity in the transformer; all measurements in that work were made on the low-tension side.

Measurement of Maximum Value of Voltage

It was at once evident from the results of the tests described above that the measurements of corona voltage would have to be made on the high-tension side of the transformer. This condition, coupled with the unreliability of the oscillograph for the higher frequencies, made it impossible to obtain complete knowledge of the wave shape, except, perhaps, by the use of the cathode ray oscillograph. It is possible, however, to measure both maximum and effective values. Obviously the former is the more important, and probably the determining factor in the formation of corona. The only method available for finding the maximum value is that used by Chubb and Fortescue,[5]

5. Chubb and Fortescue, in Proc. A. I. E. E., p. 627, Feb. 1913.

among others. The method consists essentially in applying the voltage to be investigated to a condenser and measuring the average value of the current taken by the condenser. Let V represent the maximum value attained by the voltage, C the capacity of the condenser and f the frequency. The time taken by the voltage in passing from its maximum value in one direction, to its maximum in the other, is $\frac{1}{2f}$. In this time the current goes from zero to a maximum and back to zero again. Its average value is equal to the total charge which has passed through the circuit divided by the time taken for the charge to pass. In symbols:

$$i_{av} = \frac{1}{\frac{1}{2f}} \int_0^{\frac{1}{2f}} i\,dt = \frac{2VC}{\frac{1}{2f}} = 4fVC.$$

$$V = \frac{i_{av}}{4fC} \qquad (1)$$

In order to measure the average value of the charging current taken by the condenser, it is necessary to employ a measuring instrument of the permanent magnet type and to rectify the current before it passes into the instrument, or, what amounts to the same thing, to allow only one-half of the current wave to pass through the instrument; in this latter case the reading of the instrument must be multiplied by 2 in order to get the average value of the current. Chubb and Fortescue made use of a mechanical device in order to have only one half of the current wave pass through the measuring instrument. In the present work the use of a mechanical device was out of the question on account of the high frequencies employed. The mercury arc immediately suggests itself as a non-mechanical rectifier. The rectifying properties of the mercury arc when playing between a mercury and an iron electrode are well known. The arc permits current to flow from the iron electrode to the mercury with only a trifling drop of voltage in the arc; no current can flow in the opposite direction until the applied voltage has risen so high as to cause a vacuum discharge to pass through whatever gas may be present. The mercury arc, moreover, is a very rapid in action. It has been shown, for example, that the arc will

be extinguished if the current supply is interrupted for as much as the ten-millionth part of a second. Thus the ionization or conductivity due to the passage of the current dies out with enormous rapidity.

In the present experiments the current taken by the condenser could not itself be used to maintain the arc, first, on account of its small value, and second, because of the necessary method of connection (see Fig. 2). As a consequence the cathode had to be kept active by an arc (maintained by direct current) playing between the cathode and an auxiliary mercury anode. The form of tube which was adopted after considerable experimenting is illustrated in Fig. 1. The tubes were made from German soft glass tubing having approximately an internal diameter of 1 cm. and walls 1 mm. thick. The iron electrode consisted of about 6 cm. of No. 30 iron wire which had been well oxidized by passing through the flame of a blast lamp. The object in making the iron electrode so small was to prevent mercury from condensing on it. In an earlier form of tube the iron electrode was much larger, with the result that mercury condensed on the electrode and dropped off. This last action interfered seriously with the proper functioning of the tube as, at every fall of a drop, an arc would be started, irrespective of the direction of the voltage acting across the gap. A small wire assumes the temperature of the surrounding mercury vapor much more quickly, consequently condensation is much reduced. The electrode was oxidized in order to prevent the amalgamation of the iron and the condensed mercury. The presence of such an amalgamated surface seems to facilitate condensation enormously. The tubes were exhausted while running until the pressure of the residual gas was less than 0.0001 mm.

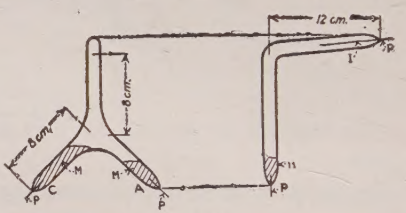

FIG. 1

P, platimum wire; I, iron wire; M, mercury; C, cathode; A, anode.

A diagram of connections is shown in Fig. 2. It will be seen that during one half of the wave the charging current of the condenser passes through one of the two tubes and during the other half through the other tube. A direct-current voltmeter was used as a current-measuring instrument. This voltmeter was in circuit with one of the tubes; consequently, in

order to get the average value of the current its reading must be multiplied by 2. The voltmeter gave full scale deflection for a current of approximately 0.01 ampere. The 1.5-volt scale (resistance approximately 156 ohms) of the instrument was used; there was no difference in reading, however, when the 150-volt scale was used instead. The instrument gave the same reading no matter with which tube it was in circuit; the reading was also unaffected by the presence of a similar instrument in series with the other tube. Each tube, moreover, was carefully tested for the degree of rectification. When the arc was playing between the two mercury electrodes a 240-volt difference of potential was applied between the mercury cathode and the iron electrode, suitable resistances being in the circuit for protection. When the iron electrode was positive with respect to the mercury cathode there was passage of current and the drop of potential across the tube was about 50 volts,

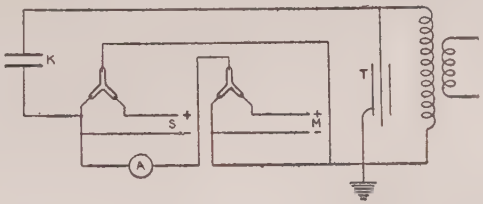

FIG. 2

A, voltmeter; S, special generator; K, condenser; M, city mains; T, corona tube.

independent of the value of current. When the iron electrode was negative with respect to the mercury cathode, the voltmeter showed no deflection and the use of a sensitive galvanometer showed that the current flowing was not greater than the five-millionth part of an ampere. As the tubes were used to rectify currents of the order of magnitude of 0.01 ampere the rectification was thus practically perfect. Finally, the tubes were set up in series with a condenser, as shown in Fig. 2, and tested at various frequencies from 45 to 75 cycles. The assumption was made that in this range of frequencies the ratio of transformation remained constant and that the wave shape of the high-tension e.m.f. was the same as that of the low-tension e.m.f. The low-tension wave form was taken from oscillograms. The values of the maximum high-tension voltage as measured directly by the mercury tubes, and as calculated from the ratio of transformation and low-tension read-

ings, were the same, within the probable error of the measurements.

Measurement of Effective Value of Voltage

While the knowledge of the maximum value of the voltage so obtained seemed of greatest importance, it was also considered desirable to determine the effective value so as to obtain an approximate indication of the wave form, and ratio of transformation. Concerning minor irregularities on the wave nothing, of course, can be said. An electrostatic voltmeter seemed to be the most suitable instrument. An instrument was needed which would give the effective value of the voltage to $\frac{1}{2}$ of one per cent or better, and which at the same time could be easily damped. These requirements suggested a Crémieu electrostatic-electrodynamic balance. The principle of this instrument is as follows: The voltage to be measured is applied to two disks, one of which is mounted at the end of one arm of a balance beam, the other disk being fixed and in proper relation to the first. The attraction between these disks is balanced by the repulsion exerted between two coils, one of which is fixed at the other end of the balance beam. A constant direct current from some suitable source is passed through these two coils, which are in series. This current is adjusted until the balance is in equilibrium in the same position that it occupies when no electric or magnetic forces are acting. The moment of the electric forces about the knife edge varies directly as the square of the voltage between the disks, and the moment of the magnetic forces varies directly as the square of the current flowing in the coils. Consequently when the balance is brought into the proper position of equilibrium the voltage between the disks is directly proportional to the current flowing in the coils. If, as in the present case, the voltage is alternating, the moment is proportional to the average square of the voltage, which is by definition the square of the effective value. Hence the effective value of the voltage is directly proportional to the current flowing in the coils.

The authors were fortunate in having available a balance beam with a fine agate knife edge, and the usual mounting. All magnetic material had been carefully excluded from the construction of the balance and its glass case. The disks were of aluminum and were 10 cm. in diameter. The distance between the fixed disk and the moving disk was 3.2 cm. The

moving coil consisted of 50 turns of No. 18 copper wire. The fixed coil had 1200 turns of No. 18 wire. The current was led into the moving balance system by means of two silver strips each 0.0013 cm. (0.0005 in.) thick, 1 mm. wide, and 6 cm. long. The points of support of a strip were 4 cm. apart, so that the strip had a very considerable sag. These silver strips would carry about 3.5 amperes before burning out. The largest current they were ever called on to carry was about 1 ampere; with this value of the current the heating of the strips was never enough to cause any disturbance of the balance due to air currents. The strips also had no effect on the sensibility of the balance, nor on the stability of the zero point. In using the balance, current for the coils was taken from a storage battery. When the voltage applied to the disks was steady, readings could be repeated to within $\frac{1}{5}$ of one per cent. When the voltage was unsteady, differences as great as one per cent frequently occurred. The balance was standardized at low frequencies, using the ratio of transformation and low-tension wave form.

Influence of Temperature and Pressure

The influences of pressure and of temperature upon the corona have not been investigated for frequencies above the neighborhood of 60 cycles. While it does not appear probable from the theory of ionization by collision that these influences would suffer any change up to 3000 cycles, it is nevertheless a matter of some uncertainty. Therefore, in order to determine the influence of frequency alone, it was necessary to make observations over the whole range of frequency within a period of time during which the pressure and temperature remained reasonably constant. Three hours was adopted as the length of such a period. This procedure made rather small the number of different frequencies at which it was possible to make observations, but the number of points obtained was sufficient to give a good knowledge of the variation with frequency. In order to shorten the time required for each reading only the maximum of the high-tension voltage wave was read when making observations on the corona. The effective values corresponding to the maximum voltages used were determined after the corona readings, thus greatly facilitating the work as the effective values of all the different voltages used over the whole range of frequencies were readily determined in a

day. Referring again to Fig. 2, the voltage on the low-tension side of the transformer was read by a Hartmann and Braun hot-wire voltmeter, the series resistance of which was specially wound to avoid change in resistance with frequency. The speed of the generator was read by a carefully calibrated tachometer, accurate within $\frac{1}{2}$ to 1/10 of one per cent, depending on the part of the scale used.

Calibration of Condensers

The condensers used were of the Moscicki type; the dielectric was glass and the condensers were filled with oil to avoid corona at the edges of the metallic coatings. Each condenser had a capacity of about 0.002 microfarad. Now the range of frequency used was from 60 to 3000 cycles, a 50-fold increase. A glance at equation (1) will show that if V stays approximately constant over this range (as it actually does) iav will be proportional to the frequency for constant capacity. This state of affairs would mean an enormous variation of the accuracy with the frequency. In order to avoid such a condition the capacity must be varied with the frequency in such a way as to keep the needle of the current-measuring instrument in a suitable part of the scale. This was done by using one condenser at 60 cycles, seven condensers in series for the range from 300 to 600 cycles, and all sixteen condensers in series for the range from 1500 to 3000 cycles. Even with the capacity reduced as much as possible by this means, it was impossible with the larger sizes of wires to go much beyond 1500 cycles, because the rectified current became larger than could be measured by the voltmeter employed; the use of a shunted instrument was, of course, out of the question.

The corona voltage at 60 cycles has been often observed; consequently it was sufficient in the present work to determine how the corona voltage varied with frequency. To this end only the *relative* values of the capacities of the condensers need be known. A special motor-driven generator, excited by a storage battery, furnished alternating current at about 45 cycles. The voltage was measured by an electrodynamometer-type voltmeter. The frequency was measured by a Hartmann and Braun vibrating-reed frequency meter. Condenser No. 1 was put in position in the connections of Fig. 2 and the low-tension voltage, frequency, and rectified current read. From seven to nine observations were made on each condenser. Then,

keeping the frequency, excitation, and all other conditions the same, No. 1 condenser was disconnected and No. 2 connected in its place; then No. 3, and so on for all sixteen condensers. Since the condensers were of approximately the same capacity and the conditions were kept constant, the maximum value of the high-tension voltage was directly proportional to the effective value of the low-tension voltage, and consequently the value of low-tension voltage could be substituted for V in the formula

$$C = \frac{i_{av}}{4 f V}$$

in order to find the comparative values of the capacities. The capacity of condenser No. 1 was taken as the unit. The capacity of the condensers when connected in series could be calculated by the usual formula. The needle of the direct-current instrument vibrated with the frequency of the rectified current, the amplitude being about one-half of a division; on this account the accuracy of the reading was only $\frac{1}{2}$ per cent. The frequency could be read to $\frac{1}{2}$ per cent and the alternating voltage to 0.1 per cent. The accuracy of any one reading was consequently about 0.7 per cent. Since from seven to nine readings were taken, the final accuracy attained was about 0.3 per cent. Repeated measurements on one condenser showed that this was actually the order of accuracy attained.

Sizes of Wires and Tubes

Two sizes of corona tubes were used in the course of the experiments, one having a diameter of 4.58 cm. and the other a diameter of 6.35 cm. The diameters of the wires used were as follows: 0.317 cm. (0.1249 in.), 0.234 cm. (0.0924 in.), 0.194 cm. (0.0763 in.), 0.164 cm. (0.0644 in.), 0.106 cm. (0.0418 in.), 0.089 cm. (0.0350 in.). All of these sizes were of brass, except the wire 0.194 cm. in diameter, which was of tool steel. The smaller sizes were made straight by heating with an electric current while under tension. The larger sizes were rods which needed no such treatment. Each time before being used the wire was polished with the very finest emery paper until the latter became glazed with brass. A final polish was given with tissue paper. This treatment produced a very good polish. In the earlier part of the work the tube was left open at the ends; for the most part, however, the ends were closed with insula-

ting caps. No difference between the two cases could be detected.

Accuracy of Observations

The rectified current could be read to 0.1 per cent at frequencies of 300 and more, the needle standing without vibration. The speed could be read always to $\frac{1}{2}$ per cent and generally still more closely. The capacity of the condensers had been determined to 0.3 per cent. Consequently any one observation of the maximum voltage at 300 cycles and over was accurate to 0.6 per cent. At 60 cycles the rectified current could be read to $\frac{1}{2}$ per cent, the frequency (the generator was not readily accessible) to $\frac{1}{2}$ per cent, and the capacity, as before, was determined to 0.3 per cent. The accuracy was consequently at 0.8 per cent. For convenience, the current required to deflect the direct-current voltmeter one scale division, and the capacity of condenser No. 1, were chosen as units of observation. The formula for the maximum voltage then becomes

$$V = k \frac{E}{f C} \qquad (2)$$

k being a constant of the apparatus and E the reading of the voltmeter. It was assumed, on the basis of information from the Bureau of Standards, that the capacity of the Moscicki condenser varies only a fraction of one per cent over the range of frequency investigated.

The Observations

It was possible in the three-hour period mentioned above, to make one or sometimes two sets of observations at 60 cycles, two sets in the neighborhood of 600 cycles and several sets between 1500 cycles and the maximum frequency obtainable as limited by the rated speed of the machine or more often by the capacity of the direct-current instrument. Each set of readings consisted of from five to twelve observations of the corona voltage. As a general thing, repeated sets of observations agreed very closely, that is, within a few tenths of one per cent, but sometimes they would differ by as much as 2 per cent. These differences could not be accounted for. The constancy of the corona voltage was not always such that full advantage could be taken of the accuracy of the method, and apparent constancy of conditions. The following table is an

example of a set of readings in which the agreement was very good:

D-C. Instrument	Low-tens. voltage of trans.	Speed in rev. per min.
140.2	102.3	968
140.5	102.3	968
140.0	102.0	973
140.0	102.7	968
140.0	102.1	968
140.0	102.1	
139.5	102.1	969
139.3	101.5	
139.8	102.1	968

The following table is an example of a rather bad set:

D-C. Instrument	Low-tens. voltage of trans.	Speed in rev. per min.
89.0		1050
91.0	Off scale	1038
90.8		1038
91.0	*i. e.*, over 130 volts.	1038
90.3		1038
90.9		1038
91.9		1037

The results for the tube 4.58 cm. in diameter are given in Table I, and those for the 6.35-cm. tube are given in Table II. The voltages given were calculated by means of formula (2). These results are plotted in Figs. 3 and 4 respectively. In these figures the points of each set are connected by straight lines in order to make interpretation easier. Each point represents a series of from five to twelve observations.

Table III gives the values of effective high-tension voltages, ratio of transformation, and peak factor, that is, ratio of maximum to effective high-tension voltage, for various voltages and frequencies over the entire range explored. The maximum values are those observed with the condenser and mercury-arc rectifier apparatus as described. The peak factor at 60 cycles was about 1.48. The range of peak factors obtained in the course of the observations consequently is from 1.66 to 1.27. The wave of voltage, therefore, did not depart greatly from

TABLE I

CORONA VOLTAGES FOR WIRES IN TUBE 4.58 CM. IN DIAMETER.

Diam. of wire	Series of observations	Frequency in cycles per second	Voltage / k
0.317 cm. (0.1249 in.)		59.0	1.242
		1338	1.191
		1680	1.202
0.234 cm. (0.0924 in.)	1	59.2	1.079
		558	1.061
		575	1.063
		1368	1.048
		1372	1.060
		1914	1.042
		1948	1.043
	2	58.5	1.081
		583	1.039
		1362	1.037
		1370	1.053
		1826	1.037
		1968	1.047
	3	58.0	1.076
		585	1.040
		1368	1.038
		1672	1.043
		1970	1.041
		2076	1.042
0.194 cm. (0.0763 in.) (tool steel)	1	59.5	0.998
		569	1.960
		2006	0.976
	2	58.8	0.999
		566	0.977
		567	0.973
		1336	0.986
		1884	0.982
	3	60.5	0.995
		590	0.950
		590	0.951
		1370	0.968
		1410	0.963
		1692	0.967
		2058	0.967
		2122	0.958
0.164 cm. (0.0644 in.)	1	59.6	0.904
		568	0.877
		576	0.893
		1376	0.895
		1418	0.888
		1754	0.885
		2176	0.884
		2412	0.880
	2	58.9	0.919
		587	0.882
		1378	0.886
		1396	0.886
		1730	0.878
		2110	0.880
		2372	0.875
	3	58.3	0.928
		584	0.880
		1368	0.885
		1674	0.894
		2054	0.890
0.106 cm. (0.0418 in.)	1	59.5	0.758
		573	0.738
		575	0.738
		2640	0.737
		2650	0.739
		2822	0.728
	2	59.1	0.748
		598	0.728
		598	0.729
		1416	0.716
		2134	0.725
		2908	0.700
		2966	0.700
0.089 cm. (0.0350 in.)	1	59.5	0.689
		60.0	0.683
		558	0.686
		570	0.680
		2822	0.666
	2	59.4	0.683
		573	0.683
		1406	0.680
		2792	0.677

the sine shape. The ratio of transformation of the transformer increased between 60 cycles and 2958 cycles by about 30 per cent.

TABLE II
CORONA VOLTAGES FOR WIRES IN TUBE 6.35 CM. IN DIAMETER.

Diam. of wire	Series of observations	Frequency in cycles per second	Voltage / k	Diam. of wire	Series of observations	Frequency in cycles per second	Voltage / k
0.317 cm. (0.1249 in.)	1	59.8	1.370	0.164 cm. (0.0644 in.)	1	58.6	0.994
		578	1.313			585	0.955
		1294	1.338			1372	0.961
		1622	1.313			1740	0.952
	2	59.4	1.362			2102	0.959
		274	1.402		2a (without series inductance in low-tension circuit.)	272	1.031
		346	1.373			274	1.037
		423	1.361			347	1.031
		502	1.366			473	0.982
		566	1.360			528	1.016
		1298	1.350			584	1.004
		1440	1.350				
		1638	1.336				
0.234 cm. (0.0924 in.)	1	58.3	1.198		2b (with series inductance in low-tension circuit.)	275	1.039
		585	1.147			346	1.019
		1360	1.148			388	0.996
		1684	1.161			423	0.982
	2	58.9	1.186			463	1.010
		284	1.223			528	1.011
		316	1.212			570	1.019
		354	1.191	0.106 cm. (0.0418 in.)	1	58.7	0.796
		467	1.091			59.4	0.795
		529	1.166			594	0.765
		566	1.191			1388	0.776
		570	1.181			1766	0.770
		1098	1.166			2134	0.775
		1460	1.170			2644	0.761
		1848	1.151				
	3	58.1	1.175				
		58.6	1.191				
		274	1.191				
		342	1.197				
		421	1.164				
		469	1.122				
		527	1.159				
		570	1.174				
		1392	1.174				

DISCUSSION

Referring to Figs. 3 and 4, it will be noticed that there is a considerable amount of irregularity in the locations of the successive points with reference to smooth curves. This

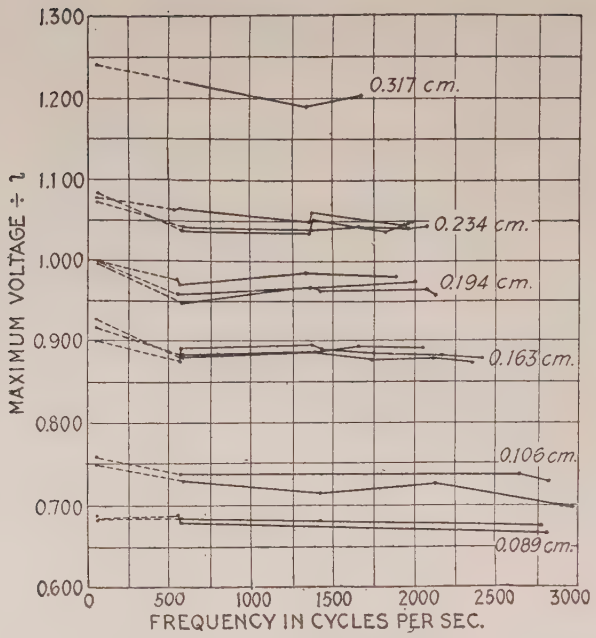

FIG. 3—CORONA VOLTAGES. TUBE 4.58 CM. IN DIAMETER.

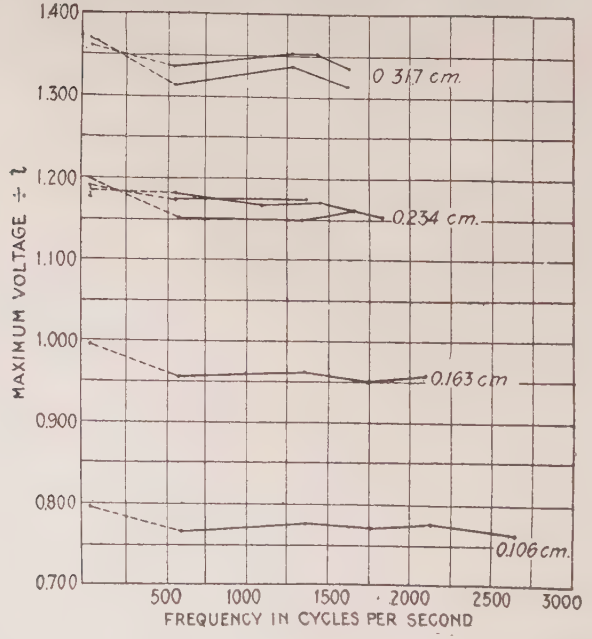

FIG. 4—CORONA VOLTAGES: TUBE 6.35 CM. IN DIAMETER.

result has been a surprise and a disappointment. Much care was taken to obtain the best conditions of accuracy in all of the instruments used. Further, during any one set of observations, there was never any indication of varying conditions. Consecutive readings taken in any one set of conditions repeated themselves accurately for the most part, and in altering the frequencies there was no suggestion, during the observations, of unstable conditions (except in one region, as described below). No cause could be found for the difference between the obser-

TABLE III

RATIO OF TRANSFORMATION AND PEAK FACTOR

Frequency in cycles per second	Low-tension voltage	Ratio of trans. R	Peak factor P	Frequency in cycles per second	Low-tension voltage	Ratio of trans. R	Peak factor P
58.4	90.6	1.000	1.000				
592	54.0	2×1.049	0.860	58.0	89.9	1.000	1.000
594	40.1	2×1.048	0.869	274	125.5	1.007	1.120
1382	62.0	1.033	0.920	274	115.5	1.007	1.120
1370	93.4	1.040	0.946	350	101.5	1.012	1.028
1732	92.6	1.074	0.920	344	130.0	1.017	1.078
2136	50.2	1.100	0.913	417	130	1.017	1.028
2112	73.2	1.131	0.921	427	53.0	0.991	1.037
2080	94.0	1.148	0.923	471	37.1	2×1.027	0.907
2392	46.5	1.189	0.912	458	65.8	2×1.011	0.888
2264	78.5	1.158	0.923	522	75.5	2×1.014	0.889
2958	44.9	1.310	0.908	526	51.5	2×1.031	0.885
				526	32.5	2×1.052	0.881
				558	74.5	2×1.038	0.894
				556	61.0	2×1.032	0.896
				560	46.9	2×1.021	0.896
				556	33.3	2×1.037	0.900

P = peak factor of 60 ~ generator = 1.48 approx.
R = ratio of transformation at 60 ~ = 125.1 : 1 or 250.2 : 1.

vations over the same range from one day to another. Temperature and pressure were eliminated as possible causes for these differences by successive series of observations under the same conditions, approximately, as to temperature and pressure.

There seems, however, to be no doubt that there is a decrease in the corona voltage, amounting to about 3 or 4 per cent, in increasing the frequency from 60 cycles to 2000 cycles or over. This decrease is apparently more pronounced for the larger sizes of wire.

It will be noticed that the points showing most serious departure from a regular curve are those for frequencies within the range 300 to 600 cycles. The observations in this range have caused much trouble and for this reason the curves have been drawn in dotted lines indicating uncertainty as to the observations. As nearly as could be determined, there was a tendency for the maximum corona voltage to take a sharp dip in passing through this range. There were, however, several accompanying conditions of experiment which may well account for the irregular results. For example, the connection of the high-tension winding of the transformer had to be altered in this range in order to make the necessary change in the corona voltage. Attention has already been drawn to the influence of this change on the charging current of the windings. It will be seen from Table III that there is evidence of a difference in the high-tension wave form for the series connection and the parallel connection of the two halves of the high-tension winding. Further, at about 425 cycles, and within a certain range of field excitation of the generator, there was a pronounced evidence of resonance in the high-tension circuit. On the smallest possible change in the field current of the generator, the reading of the d-c. instrument, indicating the maximum voltage, increased sharply from 35 to 80. Observations taken at this frequency on a larger wire and for values of voltage above the unstable region, showed no unusual conditions other than a slight irregularity of voltage which was readily damped out by series inductance in the low-tension circuit.

Still further evidence of abnormal conditions in the range 300 to 600 cycles was found in the different results obtained in the series and parallel connections of the high-tension winding Between 270 and 425 cycles with the series connection a spark would pass between wire and outer tube at a voltage only about $\frac{1}{2}$ that required to produce corona when the high-tension winding was connected in parallel. The same phenomenon was observed in the range 450 to 600 cycles, with the exception that the parallel connection gave the spark at low voltage. This spark was intermittent, apparently localized, and very different from the usual uniform corona. The impression made was that there was a series of short impulses of abnormally high voltage.

While it is evident, therefore, that there were a number of abnormal conditions surrounding the observations in the region 300 to 600 cycles, there is no obvious reason why, if the conditions

were steady, the maximum value of voltage could not have been picked out by the mercury arc peak voltmeter. There is every indication that the mercury arc is instantaneous in action. Nevertheless, there seems to be no other conclusion than that the conditions of resonance described above introduced very brief voltage impulses of high value which the mercury arc and condensers did not pick out. Considerable variation was obtained in the wave form by connecting the armatures of the two generators in series, thus obtaining a prounounced fifth harmonic. There was no resulting variation of the corona voltage; such variation, however, could hardly be expected unless the frequencies were far higher than 3000 cycles.

The corona from 1500 cycles upward appears also to differ qualitatively from that at lower frequencies. It is a common observation at 60 cycles that the gold-leaf of the electroscope descends in a jerky fashion when corona voltage is reached. The time taken for complete discharge is about 30 seconds. This has been attributed to the exact constancy of the corona voltage and the usual slight variation of the impressed voltage. From 300 to 600 cycles the electroscope discharged quickly but steadily, the time for discharge being one or two seconds. From 1500 to 3000 cycles the electroscope discharged in a very small fraction of a second, the gold-leaf descending practically instantaneously. There is absolutely no indication of leak before any of these observations.

A further interesting phenomenon at the higher frequencies was the vibration of the wire in the case of the open tube, when the points of suspension were some distance beyond the ends. At 1500 cycles the presence of corona caused the wire to vibrate greatly. There was no vibration before the appearance of the corona. The tension on the wire was altered through a wide range, but the vibration persisted unchanged. There is apparently, therefore, no connection between the frequency of the voltage and the natural period of vibration of the stretched wire. At 60 cycles the wire remained perfectly steady, even when surrounded by a strong corona. These vibrations, therefore, are apparently forced vibrations due to some property of the corona.

In considering the possible influence of the frequency on the appearance of corona in terms of the theory of secondary ionization or ionization by collision, there are three possible ways in which this influence might act: (1) The conditions of stable equilibrium

of the systems of vibrating electrons within the atom of the gas would be affected in different degrees by periodic electric forces of different frequencies. (2) With increasing frequencies the time during which a single free moving electron is acted on by the electric force in one direction would be less; consequently the velocity acquired and energy available for ionization by collision would be less. (3) It has been suggested that the corona occurs when the density of ionization in the gas at the surface of the wire reaches a certain value. This increase in density results from the passage inward of electrons from a distance. As they move in they create more ions by collision. More ions are created on the inward path than are lost by recombination during the reverse half of the alternating cycle. At higher frequencies the rate at which the ions would reach the wire would probably be lowered, thus demanding a higher voltage for the appearance of corona.

Of these three possible methods of explanation the first two are apparently untenable, certainly for the range of frequencies investigated in these experiments, by reason of the fact that the periods of vibration within the atom and the time during which an electron passes from one collision to another, are infinitesimally small compared with the time of reversal of the electric force at the frequencies investigated.

The experiments were carried out in the Physical Laboratory of the Johns Hopkins University. The authors wish to acknowledge the valuable assistance of Professor A. H. Pfund in connection with the mercury lamps and of Mr. M. W. Pullen at various times throughout the work.

Summary

The most important results of the work are as follows:

(1) The corona voltage at 2000 cycles is lower by 3 or 4 per cent than it is at 60 cycles.

(2) The corona in the range 300 to 3000 cycles has a number of qualitative differences from that at 60 cycles.

(3) Convenient methods for measuring high-tension maximum and effective voltages at frequencies up to 3000 cycles have been devised.

(4) An instance of resonance due to the capacity of the high-tension windings of a transformer is recorded.

Biographical Sketch

William Stuart Gorton was born in Baltimore, Maryland, on March 28, 1889. He received his preparatory education in the public schools of Baltimore, graduating from the Baltimore City College in 1905. In the fall of that year he entered The Johns Hopkins University, taking the Mathematical-Physical group of studies. He held a Hopkins Scholarship for three years and received the degree of Bachelor of Arts in 1908. In October of that year he entered the Graduate Department, taking as his subjects Physics, Applied Electricity, and Mathematics. He held a University Scholarship during the year 1908-1909 and in the year 1909-1910 he was Student Assistant in the Physical Laboratory. He received the degree of Master of Arts in 1910 and was Fellow in Physics during 1910-1911. From 1911 to 1913 he was Instructor in Electrical Engineering at the Massachusetts Institute of Technology. During the year 1913-1914 he was Fellow by Courtesy at The Johns Hopkins University.